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***Abstract*— In the railway technical centers, scheduling the maintenance activities is a very complex task, it consists in ordering, in the time, all the maintenance operations on the workstations, while respecting the number of resources, precedence constraints, and the workstations' availabilities. Currently, this process is not completely automatic. For improving this situation, this paper presents a mathematical model for the maintenance activities scheduling in the case of railway remanufacturing systems. The studied problem is modeled as a flexible job-shop, with the possibility for a job to be executed several times on a stage. MILP formulation is implemented with the Makespan as an objective, representing the time for remanufacturing the train. The aim is to create a generic model for optimizing the planning of the maintenance activities and improving the performance of the railway technical centers. At last, numerical results are presented, discussing the impact of the instances size on the computing time to solve the described problem.**

Keywords—maintenance scheduling, flexible job-shop, railway system, Makespan, optimization.

I. Introduction and Literature Review

To ensure the functioning of rolling stock, the railway companies have several maintenance technical centers. These technical centers are classified into two categories. The first one is the maintenance technical centers, in which is performed the maintenance of level 1 to 3. The second one is Industrial Technical Centers (ITC) in which is performed the heavy maintenance operations, modernization and remanufacturing the trains, these are the maintenance operations of level 4 and 5. In fact, most of the operations carried out at ITC are midlife operations on TGVs (High Speed Train). After 15 years of use, TGVs are sent to ITC to be completely dismantled to undergo a series of checks and repairs before being reassembled and modernized. These operations take about 4 months of working, they represent about 60000 hours of workload.

The maintenance activities include preventive maintenance and renewal. The former includes activities that restore the train components to a better condition. The latter is the replacement of components when maintaining them is no longer practical and economical [1]. In the case of ITC, a train is composed by several wagons, each wagon represents a job and each job is made up of several operations. The operations represent the elementary work carried out on each workstation. For each operation, a duration representing its workload and a workstation in which it can be executed. Some workstations are occupied by several operators who can make the same job. From modeling point of view, this system can be considered as a flexible job-shop for the following reasons: A chain of operations must be executed on workstations in series. A train is composed by several wagons, each wagon represents a job. A job includes several operations, and each operation must be executed on a workstation representing the stage. Each job follows its own sequence on the stages. Some operations can simultaneously be executed on identical workstations when multiple operators are available. So, there are parallel resources in some stages. The schedule then determines the ordering of the wagon's operations on the workstations for minimizing the time for remanufacturing the train, regarded as a makespan.

In the railway system literature, several works modeled the planning of maintenance activities as a scheduling problem. For example, Peng et *al.*, [2] proposed a heuristic method to the railroad track maintenance scheduling. The authors are interested in the minimization of the total travel costs of the maintenance teams as well as the impact of maintenance projects on railroad operation. Budai et *al.*, [3] are interested in the long-term and medium-term planning for determining which preventive maintenance works will be performed on segments and its time periods. The aim is to minimize the track possession cost. The authors proposed three greedy heuristics for scheduling preventive railway maintenance activities. Consilvio et *al.*, [4] presents a stochastic model for scheduling predictive and risk-based maintenance activities in rail sector, the authors considered the stochastic nature of railway defect occurrence. They developed a MILP formulation for modeling their problem, considering the completion time and the tardiness of the maintenance activities as an objective. Dao et *al.*, [5] proposed a mathematical model for finding the best maintenance schedule for multiple components in a railway track for minimizing the total cost in the planning horizon. For other related papers dealing with railway maintenance activities scheduling, the reader may refer to [6], [7], [8], [9], [10].

Most of studied works consider the scheduling of routine or ordinary maintenance activities, such as regular inspections and minor repairs. However, only few papers are interested in scheduling major maintenance activities like renewal, modernization, or remanufacturing. For instance, Torba et *al*., [11] proposed a MILP formulation for scheduling maintenance operations of rolling stock to renovate the trains. The authors modeled their problem as a multi-skill resource-constrained muti-project scheduling. To fill the void, in this paper, a MILP formulation for scheduling the maintenance activities of level 4 and 5 is implemented. The studied problem is very helpful for ITC decision makers. It allows to optimize the time of remanufacturing the train and reduce the related costs. In the classical job-shop, each job consists of a set of operations which need to be proceeded in a specific stage and only one operation in a job can be proceeded at a given time. Each job follows its own sequence on the stages. When there are multiple machines on the stage, we are talking about a flexible job-shop. In the studied problem, some jobs can be proceeded several times on a stage. For example, a wagon proceeds on the washing stage, then it proceeds on sanding, and it comes back to the washing again. Therefore, in our model, the jobs can proceed several times on a stage, but not successively. The novelty of this work is to model the maintenance activities scheduling of railway remanufacturing system (maintenance of level 4 and 5) as a flexible job-shop scheduling problem and to create a generic mathematical model using a MILP formulation. To the best of our knowledge, very few papers consider this kind of problem. The main contributions of the work are:

- Modeling the maintenance activities of the railway remanufacturing systems as a flexible job-shop scheduling problem.
- Implementing a MILP formulation for the described problem to optimize the time of remanufacturing the train.
- Discussing the results in terms of the *CPU* time for testing the limitation of the MILP resolution in terms of number of jobs.

The rest of the paper is organized as follows: In section 2, the problem is described. In section 3, the mathematical model is implemented. In section 4, the numerical results are discussed. Finally, we conclude and propose some perspectives in section 5.

II. Problem description

In the studied system, a train is composed by several wagons, each wagon represents a job, each job is made up of several operations. The operation represents the elementary work carried out on each workstation. For each operation, a duration representing its workload and a workstation in which it can be executed. Indeed, a workstation is a physical place where the operation can be performed. Some workstations are occupied by several operators who can make the same job. From modeling point of view, this system can be considered as a flexible job-shop. For the reasons mentioned in the introduction.

In the flexible job-shop problem, we consider a set of N jobs, $\{1, 2, \ldots, N\}$ is available to be scheduled, at the beginning of the horizon, in a set of S stages, $\{1, 2, \ldots, S\}$. Each job is composed by several operations, we denoted by O_{ij} the i^{th} operation of job j. Thus, for each operation O_{ij} is given a processing time p_{ij} and a stage $Stage_{ij}$. When the operation O_{ij} starts at time ST_{ij} on a stage $Stage_{ij}$, it is processed to completion time CT_{ij} without preemption (i.e. $CT_{ij} = ST_{ij} + p_{ij}$). All the job information (p_{ij}, $Stage_{jij}$) is known in advance. The Makespan C_{max} is the completion of all operations of the train, *max* (CT_{ij}).

III. Mathematical model

In this section, a MILP is formulated. To build our mathematical model, we adapted the model presented in [12,13,14,15] in which they minimized the total weighted waiting time in scheduling problems, to consider now the flexible job-shop and the makespan as an objective. The assumptions, parameters, decision variables and constraints are given below

Assumptions

- The maintenance operators are always available to perform the tasks.
- The technical center is continuously open.
- The dismantling operations are done before.
- The operations scheduling concerns the remanufacturing of one train.

Parameters

N: number of jobs (wagons)
S: number of stages (workstations)
j: index of job, j=1, 2, ..., N
k: index of position, k=1, 2, ..., K
s: index of stage, s=1, 2, ..., S
M_s: number of identical machines in the stage s.
m: index of machine at stage s, m=1, 2, ..., M_s.
n_j: number of operations of job j.
i: index of operation, i=1, 2, ..., n_j
O_{ij}: i^{th} operation of job j.
p_{ij}: duration of the operation O_{ij}.
$Stage_{ij}$: stage the operation O_{ij}.
$BigM$: A big value. In our case, it is assumed that $BigM$= $\sum_{j=1}^{N}\sum_{i=1}^{n_j} p_{ij}$
A_{js}: number of times when job j must be executed in the stage s.
K: $\max\limits_{s\ in\ 1..S} \sum_{j=1}^{N} A_{js}$

Decision variables

ST_{ij}= starting time of operation i of job j.
CT_{ij}= completion time of operation i of job j.
S_{ksm}= starting time of position k in stage s at machine m.
C_{ksm}= completion time of position k in stage s at machine m.

$$X_{ijksm}\begin{cases}1 & if\ operation\ O_{ij}\ assigned\ in\ position\ k\ in\ stage\ s\ on\ machine\ m \\ 0 & otherwise\end{cases}$$

Objective

The objective is to minimize the schedule completion time, the completion of all operations of the train. Hence, we reduce the costs related to the train delay. This date corresponds to the Makespan C_{max} such as:

$$C_{max} = max\left(CT_{ij}\right)\ \forall i \in \{1 \ldots nj\}, \forall j \in \{1 \ldots N\}$$

So, the objective is min C_{max} subject to the following constraints.

Constraints

$$C_{max} \geq CT_{ij}, \ \ \forall i \in \{1 \dots nj\}, \forall j \in \{1 \dots N\} \quad (1)$$

$$\sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{Ms} X_{ijksm} = 1, \ \ \forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\} \quad (2)$$

$$\sum_{k=1}^{K} \sum_{m=1}^{Ms} X_{ijksm} = 1, if \ (Stage_{ij} = s)$$
$$\forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\}, \forall s \in \{1 \dots S\} \quad (3)$$

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} X_{ijksm} \leq 1; \forall k \in \{1 \dots K\}$$
$$\forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (4)$$

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} X_{ijk+1sm} \leq \sum_{j=1}^{N} \sum_{i=1}^{n_j} X_{ijksm}$$
$$\forall k \in \{1 \dots K-1\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (5)$$

$$S_{k+1sm} \geq C_{ksm}, \forall k \in \{1 \dots K-1\},$$
$$\forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (6)$$

$$C_{ksm} = S_{ksm} + \sum_{j=1}^{N} \sum_{i=1}^{n_j} X_{ijksm} * p_{ij}$$
$$\forall k \in \{1 \dots K\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (7)$$

$$CT_{ij} = ST_{ij} + p_{ij}, \ \ \forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\} \quad (8)$$

$$CT_{ij} \geq C_{ksm} - BigM(1 - X_{ijksm}),$$
$$\forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\}, \forall k \in \{1 \dots K\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (9)$$

$$CT_{ij} \leq C_{ksm} + BigM(1 - X_{ijksm}),$$
$$\forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\}, \forall k \in \{1 \dots K\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (10)$$

$$CT_{ij} \leq ST_{i'j} \ if \ (i < i'), \forall j \in \{1 \dots N\}, \forall i, i' \in \{1 \dots n_j\} \quad (11)$$

$$X_{ijksm} \in \{0,1\},$$
$$\forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\}, \forall k \in \{1 \dots K\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (12)$$

$$CT_{ij}, ST_{ij}, C_{ksm}, S_{ksm} \geq 0$$
$$\forall j \in \{1 \dots N\}, \forall i \in \{1 \dots n_j\}, \forall k \in \{1 \dots K\}, \forall s \in \{1 \dots S\}, \forall m \in \{1 \dots M_s\} \quad (13)$$

Constraints' description

Equation (1) means that the Makespan value must be greater than or equal to all completion time of positions. Equation (2) consists of having one operation per position. Equation (3) consists of assigning each operation to a stage. Equation (4) consists of having only one or no operation per position in a machine in a stage. Equation (5) ensures that if position k is not occupied by a job, position k+1 will not be occupied either. Equation (6) consists in making, for all machines, the starting time of the $(k+1)^{\text{th}}$ position greater than or equal to the previous position completion time. Equation (7) specifies that, for all machines, the completion time of the k^{th} position is equal to the starting time of the k^{th} position plus the assigned job processing time. Equation (8) specifies that, the completion time of O_{ij} is equal to the starting time of O_{ij} plus its processing time. Equation (9) and (10) defines, for all machines and all stages, the completion time of operation O_{ij}, which is greater than or equal to the completion time of the assigned position, where *bigM* must be sufficiently large. Equation (11) making the starting time of operation $O_{i'j}$ greater than the completion time of operation O_{ij} if i' is after i. Equation (12) defines the domain of the variable X_{ijksm}. Equation (13) is no negativity constraint.

In this formulation, the blocking constraints between successive stages are not considered. The blocking constraints can eventually be added in the equation (6), modeling the case where the buffer space capacity between the machines is limited.

IV. Experimental results

The proposed MILP model has been written on FICO Xpress IVE and the simulations have been performed on a Core i7 2.70GHz laptop. In this section, we present the numerical results obtained through the model application on a flexible job-shop. The considered processing times vary following a normal distribution with an average of 30 hours and a standard deviation of 50 hours, $p_{ij} \sim N\,(30, (50)^2)$. For testing the limitations of the model, we varied the number of jobs N, with different values (3, 5; 10), Sg = (5;10;15). At each stage is executed one operation by one machine. On the presented results, we tested on 10 different instances the computing time (*CPU*). Then, we calculated the minimum, maximum, average, and standard deviation of *CPU* in second. The obtained results are presented in Table 1.

TABLE I. *CPU* VALUES

Sg	***N***	**Min *CPU***	**Max *CPU***	**Avg *CPU***	**Std dev *CPU***
5	3	0.125	0.157	0.141	0.02
	5	7.815	8.96	8.38	0.8
	10	950	3200	1123.2	1203
10	3	0.602	0.741	0.6715	0.09
	5	29.281	33.11	31.19	2.7
	10	> 12h	> 12h	> 12h	> 12h
15	3	2.09	2.29	2.19	0.14
	5	> 12h	> 12h	> 12h	> 12h
	10	> 12h	> 12h	> 12h	> 12h

The Average *CPU* increases to much with N and Sg. Thus, the average *CPU* depends on both, the number of jobs (wagons) and the number of stages (workstations). If these ones become large, the average and the standard deviation of *CPU* increase. This matter makes it difficult to estimate the necessary computing time that the MILP needs for solving the problem. Based on Table 1, the proposed MILP model is effective in solving the problems made up of 5 jobs in 10 stages, or 3 jobs in 15 stages. It is averagely equivalent to 50 operations in total. We have also performed the problems for 5 jobs in 15 stages (75 operations). The program ran for 12 hours. Then, we have interrupted the simulation.

V. Conclusions and perspectives

This paper proposed a MILP formulation for scheduling the maintenance activities of level 4 and 5. The problem is inspired from the railway remanufacturing systems, in which the system is modeled as a flexible job-shop scheduling, with the possibility for a job to be executed several times on a stage. The Makespan is considered as an objective, representing the time for remanufacturing the train. The aim of the study is to create a generic model for optimizing the planning of the maintenance activities and improving the performance of the technical centers. Numerical results show that the MILP model is effective in solving the problems made up of 50 operations. To solve large real industrial instances, one of the main short-term prospects is to design efficient heuristics and metaheuristics to browse even more jobs in a reasonable time, as well as a comparison of the proposed MILP and the future heuristics with other existing resolution methods. We will also consider some future aspects that are beyond the scheduling of operations, such as:

- The precedence constraints and the starting conditions between the phases.
- The dismantling assembly operations.
- The workshop opening hours.

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REFERENCES

[1] Levy, G. (2012). "Optimal Planning of Track Work." Cost Optimization Track Maintenance and 522 Renewal Congress, London.

[2] Peng, F., Kang, S., Li, X. and Ouyang, Y. (2011). A Heuristic Approach to the Railroad Track Maintenance Scheduling Problem. Computer-Aided Civil and Infrastructure Engineering, Vol. 26, No. 22, pp. 129-145.

[3] Budai, G., Huisman, D. and Dekker, R. (2004). Scheduling Preventive Railway Maintenance Activities .Journal of Operational Research Society, Vol. 57, No. 9, pp. 1035-1044

[4] Consilvio, A., Di Febbraro, A., & Sacco, N. (2016, August). Stochastic scheduling approach for predictive risk-based railway maintenance. In 2016 IEEE international conference on intelligent rail transportation (ICIRT) (pp. 197-203). IEEE.

[5] Dao, C., Basten, R., & Hartmann, A. (2018). Maintenance scheduling for railway tracks under limited possession time. *Journal of Transportation Engineering, Part A: Systems*, *144*(8), 04018039

[6] Doganay, K., & Bohlin, M. (2010). Maintenance plan optimization for a train fleet. WIT Transactions on The Built Environment, 114(12), 349-358.

[7] Consilvio, A., Di Febbraro, A., Meo, R., & Sacco, N. (2019). Risk-based optimal scheduling of maintenance activities in a railway network. EURO journal on transportation and logistics, 8(5), 435-465.

[8] Lin, B., Wu, J., Lin, R., Wang, J., Wang, H., & Zhang, X. (2019). Optimization of high-level preventive maintenance scheduling for high-speed trains. Reliability Engineering & System Safety, 183, 261-275.

[9] Bakhtiary, A., Mohammadzadeh, S., Zakeri, J. A., & Kasraei, A. (2022). Multiobjective Optimization of Ballasted Track Maintenance Scheduling. Journal of Transportation Engineering, Part A: Systems, 148(1), 04021099.

[10] Zhang, Q., Lusby, R. M., Shang, P., & Zhu, X. (2022). A heuristic approach to integrate train timetabling, platforming, and railway network maintenance scheduling decisions. Transportation Research Part B: Methodological, 158, 210-238.

[11] Torba, R., Dauzere-Peres, S., Yugma, C., Gallais, C., & Ramond, F. (2022, February). Robust scheduling within SNCF railway maintenance centers. In 23ème congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision.

[12] Tighazoui, A., Sauvey, C., & Sauer, N. (2021). Predictive-reactive Strategy for Flowshop Rescheduling Problem: Minimizing the Total Weighted Waiting Times and Instability. Journal of Systems Science and Systems Engineering, 30(3), 253-275.

[13] Tighazoui, A., Sauvey, C., & Sauer, N. (2021). Heuristics based on MILP for solving a flow shop rescheduling problem simultaneously considering efficiency and stability. *17th International Conference on Automation Science and Engineering (CASE)* (pp. 2079-2084). IEEE.

[14] Tighazoui, A. (2021). Ré-ordonnancement des systèmes de production flexibles avec contraintes de blocage mixtes soumis à des aléas de commandes ou de production (Doctoral dissertation, Université de Lorraine).

[15] Tighazoui, A., Sauvey, C., & Sauer, N. (2021). Predictive-reactive strategy for identical parallel machine rescheduling. Computers & Operations Research, 134, 105372.